



AP[®] Calculus BC
2005 Free-Response Questions
Form B

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2005 AP[®] CALCULUS BC FREE-RESPONSE QUESTIONS (Form B)

CALCULUS BC
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

1. An object moving along a curve in the xy -plane has position $(x(t), y(t))$ at time $t \geq 0$ with

$$\frac{dx}{dt} = 12t - 3t^2 \text{ and } \frac{dy}{dt} = \ln(1 + (t - 4)^4).$$

At time $t = 0$, the object is at position $(-13, 5)$. At time $t = 2$, the object is at point P with x -coordinate 3.

- (a) Find the acceleration vector at time $t = 2$ and the speed at time $t = 2$.
 - (b) Find the y -coordinate of P .
 - (c) Write an equation for the line tangent to the curve at P .
 - (d) For what value of t , if any, is the object at rest? Explain your reasoning.
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WRITE ALL WORK IN THE TEST BOOKLET.

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2. A water tank at Camp Newton holds 1200 gallons of water at time $t = 0$. During the time interval $0 \leq t \leq 18$ hours, water is pumped into the tank at the rate

$$W(t) = 95\sqrt{t} \sin^2\left(\frac{t}{6}\right) \text{ gallons per hour.}$$

During the same time interval, water is removed from the tank at the rate

$$R(t) = 275\sin^2\left(\frac{t}{3}\right) \text{ gallons per hour.}$$

- (a) Is the amount of water in the tank increasing at time $t = 15$? Why or why not?
- (b) To the nearest whole number, how many gallons of water are in the tank at time $t = 18$?
- (c) At what time t , for $0 \leq t \leq 18$, is the amount of water in the tank at an absolute minimum? Show the work that leads to your conclusion.
- (d) For $t > 18$, no water is pumped into the tank, but water continues to be removed at the rate $R(t)$ until the tank becomes empty. Let k be the time at which the tank becomes empty. Write, but do not solve, an equation involving an integral expression that can be used to find the value of k .
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3. The Taylor series about $x = 0$ for a certain function f converges to $f(x)$ for all x in the interval of convergence. The n th derivative of f at $x = 0$ is given by

$$f^{(n)}(0) = \frac{(-1)^{n+1} (n+1)!}{5^n (n-1)^2} \text{ for } n \geq 2.$$

The graph of f has a horizontal tangent line at $x = 0$, and $f(0) = 6$.

- (a) Determine whether f has a relative maximum, a relative minimum, or neither at $x = 0$. Justify your answer.
- (b) Write the third-degree Taylor polynomial for f about $x = 0$.
- (c) Find the radius of convergence of the Taylor series for f about $x = 0$. Show the work that leads to your answer.
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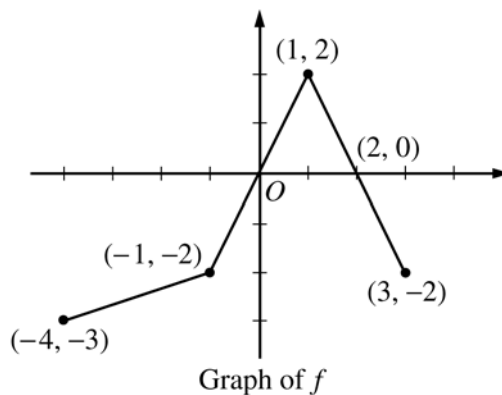
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END OF PART A OF SECTION II

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CALCULUS BC
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.



4. The graph of the function f above consists of three line segments.
- (a) Let g be the function given by $g(x) = \int_{-4}^x f(t) dt$. For each of $g(-1)$, $g'(-1)$, and $g''(-1)$, find the value or state that it does not exist.
- (b) For the function g defined in part (a), find the x -coordinate of each point of inflection of the graph of g on the open interval $-4 < x < 3$. Explain your reasoning.
- (c) Let h be the function given by $h(x) = \int_x^3 f(t) dt$. Find all values of x in the closed interval $-4 \leq x \leq 3$ for which $h(x) = 0$.
- (d) For the function h defined in part (c), find all intervals on which h is decreasing. Explain your reasoning.

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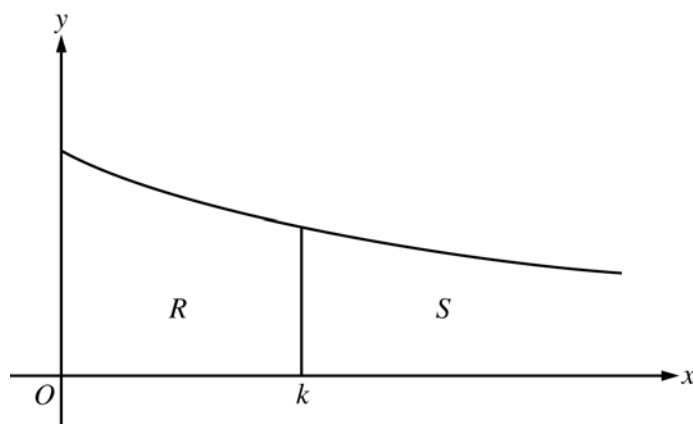
5. Consider the curve given by $y^2 = 2 + xy$.

(a) Show that $\frac{dy}{dx} = \frac{y}{2y - x}$.

(b) Find all points (x, y) on the curve where the line tangent to the curve has slope $\frac{1}{2}$.

(c) Show that there are no points (x, y) on the curve where the line tangent to the curve is horizontal.

(d) Let x and y be functions of time t that are related by the equation $y^2 = 2 + xy$. At time $t = 5$, the value of y is 3 and $\frac{dy}{dt} = 6$. Find the value of $\frac{dx}{dt}$ at time $t = 5$.



6. Consider the graph of the function f given by $f(x) = \frac{1}{x+2}$ for $x \geq 0$, as shown in the figure above. Let R be the region bounded by the graph of f , the x - and y -axes, and the vertical line $x = k$, where $k \geq 0$.

(a) Find the area of R in terms of k .

(b) Find the volume of the solid generated when R is revolved about the x -axis in terms of k .

(c) Let S be the unbounded region in the first quadrant to the right of the vertical line $x = k$ and below the graph of f , as shown in the figure above. Find all values of k such that the volume of the solid generated when S is revolved about the x -axis is equal to the volume of the solid found in part (b).

WRITE ALL WORK IN THE TEST BOOKLET.

END OF EXAM